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Concerning Lateral Dynamics of Flight on a Great Circle

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Introduction

IN a fairly recent paper, Drummond[1] considered the lateral stability of a hypersonic vehicle, representative of a space shuttle or hypersonic transport, which was flying above the Earth in a great circle. He obtained a set of six, coupled, linear differential equations with constant coefficients which govern the perturbed motion of such a vehicle.

The characteristic equation associated with the linear system of Ref. 1 is the product of quartic and quadratic factors. The roots of the quadratic factor are complex conjugates with zero real parts and hence together represent an undamped oscillatory mode. The periodic motion in this mode is a phenomenon not predicted by results of conventional stability studies which utilize a flat-Earth model and a rectilinear reference flight path.

In this Note, a simple transformation of one of Drummond's dependent variables is made with the result that the dynamics of the perturbed lateral motion are uncoupled from the kinematics of the flight path. That is, the transformation allows one to separate the governing equations into two sets of equations, one of fourth order (dynamic equations) and one of second order (kinematic equations). The dynamic equations are uncoupled from the kinematic equations, but not the converse. As such, the equations obtained here are very similar to those obtained using a flat-Earth model.[2] The principal difference is that the two kinematic equations are coupled and when perturbations in the dynamic variables vanish, the kinematic equations yield the undamped oscillatory motion cited in Ref. 1. Furthermore, the modified equations presented here allow an interesting physical interpretation of the new oscillatory mode to be made.

Analysis

If the variable,† $\chi = \phi - \bar{y}_1$ (see Fig. 1), (where ϕ is the roll perturbation angle of the vehicle, $\bar{y}_1 = y_1/R_e$, y_1 is the lateral displacement of the center of mass of the vehicle and R_e is the radius of the Earth) is introduced into Eqs. (14–19) of Ref. 1, then the following two sets of equations may be obtained:

$$d\chi/d\hat{t} = (L/b)\hat{p} - (L/2R_e)\cos\phi_o\,\beta \tag{1a}$$

$$2\mu(d\beta/d\hat{t}) = C_{y_\beta}\beta + C_{y_p}\hat{p} + (C_{y_r} - 2\mu L/b)\hat{r} + C_W\cos\phi_o\,\chi \tag{1b}$$

$$\begin{aligned} D(d\hat{p}/d\hat{t}) = {} & (i_C C_{l_\beta} + i_E C_{n_\beta})\beta + \\ & [i_C C_{l_p} + i_E C_{n_p} + 2\hat{Q}_o(L/b)^2 i_E(i_A - i_B + i_C)]\hat{p} + \\ & \{i_C C_{l_r} + i_E C_{n_r} + 2\hat{Q}_o(L/b)^2[i_C(i_B - i_C) - i_E{}^2]\}\hat{r} \end{aligned} \tag{1c}$$

$$\begin{aligned} D(d\hat{r}/d\hat{t}) = {} & (i_A C_{n_\beta} + i_E C_{l_\beta})\beta + \\ & \{i_A C_{n_p} + i_E C_{l_p} + 2\hat{Q}_o(L/b)^2[i_A - i_B + i_E{}^2]\}\hat{p} + \\ & \{i_A C_{n_r} + i_E C_{l_r} + 2\hat{Q}_o(L/b)^2[i_B - i_A - i_C]i_E\}\hat{r} \end{aligned} \tag{1d}$$

where

$$D = 2(L/b)^2(i_A i_C - i_E{}^2)$$

and

$$d\psi/d\hat{t} + (L/2R_e)\bar{y}_1 = -(L/2R_e)\chi + (L/b)\cos\phi_o\,\hat{r} \tag{2a}$$

$$d\bar{y}_1/d\hat{t} - (L/2R_e)\psi = (L/2R_e)\cos\phi_o\,\beta \tag{2b}$$

In Eqs. (1) and (2), β is the side-slip angle, $\hat{p}$ and $\hat{r}$ are the x- and z-components‡ (roll and yaw) of the perturbation in the vehicle's angular velocity, and ψ is the perturbation in the vehicle's yaw angle. Also, $\hat{Q}_o = -L\cos\phi_o/2R_e$, where L is the

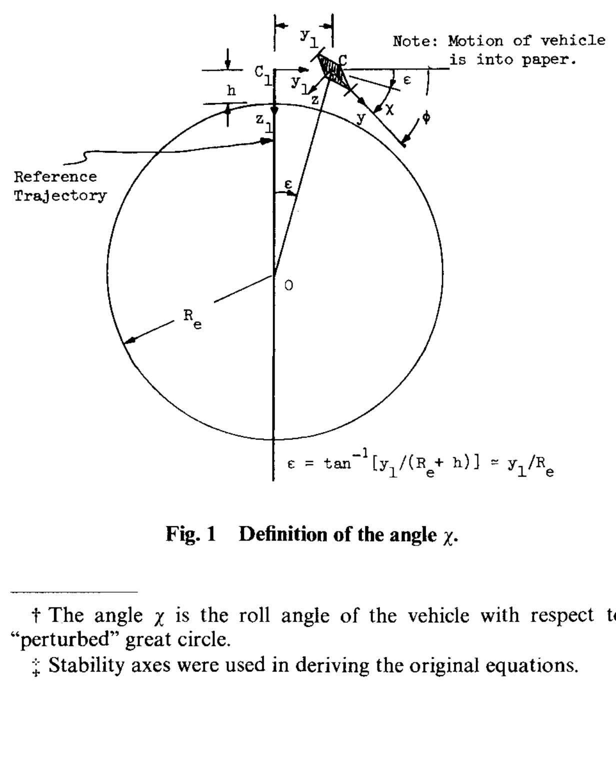

Fig. 1 Definition of the angle χ.

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† The angle χ is the roll angle of the vehicle with respect to a "perturbed" great circle.

‡ Stability axes were used in deriving the original equations.

length of the vehicle and $\phi_o = 0$ or π according to whether the vehicle's reference flight speed, U_o, is less than or greater than orbital speed at the reference flight altitude, h. Furthermore, $\hat{t} = t/t^*$, where t is the time and $t^* = L/2U_o$. To retain conciseness here, the reader is requested to refer to Ref. 1 for further definitions of the symbols used in Eqs. (1) and (2).

Note that the dynamic Eqs. (1) for χ, β, $\hat{p}$, and $\hat{r}$ are uncoupled from the kinematic Eqs. (2) for the lateral center of mass displacement and yaw perturbation angle, as are the equations for ϕ, β, $\hat{p}$, and $\hat{r}$ in the conventional flat-Earth equations.[2] Of course, Eqs. (2) are slightly more complex than their flat-Earth counterparts.

An obvious solution to Eqs. (1) and (2) is

$$\chi = \beta = \hat{p} = \hat{r} = 0 \tag{3}$$

$$\bar{y}_1 = \bar{y}_1{}^* = a_o \cos\hat{\omega}\hat{t} + b_o \sin\hat{\omega}\hat{t} \tag{4a}$$

$$\psi = \psi^* = -a_o \sin\hat{\omega}\hat{t} + b_o \cos\hat{\omega}\hat{t} \tag{4b}$$

where a_o and b_o are constants of integration and $\hat{\omega} = L/(2R_e)$. Since $\phi = \bar{y}_1 + \chi$, in this special case, $\phi = \bar{y}_1 = \bar{y}_1{}^*$. This is the new oscillatory mode cited in Ref. 1.

The exact period of the reference great-circle motion is $2\pi/\Omega$, where $\Omega = U_o/(R_e + h)$, but Drummond used the approximation $\Omega \simeq U_o/R_e$. The period of the oscillatory motion defined by Eqs. (4) is also $2\pi R_e/U_o$.

By referring to Fig. 2 and recalling that $\bar{y}_1 = y_1/R_e$, it is easily seen that the motion corresponding to Eqs. (4) and $\phi = \bar{y}_1$ is *steady flight* in a great circle which is rotated with respect to the original one through an angle, $I = (a_o{}^2 + b_o{}^2)^{1/2}$. The maximum value of $\bar{y}_1{}^*$ (and ϕ) occurs at point A of Fig. 2 and the maximum ψ^* occurs at point B.

It should be noted that if the approximations $U_o/(R_e + h) \simeq U_o/R_e$, $g = g_o/\{1 + [\bar{y}_1/(1+h)]^2\} \simeq g_o$, and $\varepsilon = y_1/(R_e + h) \simeq y_1/R_e$ had not been used in Ref. 1, then if a "free-satellite" condition happened to exist, the period of the oscillatory mode would be exactly equal to the orbital period.

Suppose that perturbations in the variables, χ, β, $\hat{p}$, and $\hat{r}$, occur at $\hat{t} = 0$. Suppose also that the solution to Eqs. (1) is asymptotically stable. It then follows, by using Eqs. (2), that the resulting steady-state motion of the vehicle will, in general, correspond to motion on a "perturbed" great circle, which is rotated with respect to the reference great-circle, and the vehicle ends up in the undamped oscillatory mode. The displacement of the steady-state trajectory in this case is quite analogous to that predicted by flat-Earth lateral stability analyses.[3]

The equations given here may be easily reduced to their flat-Earth counterparts. If the flat-Earth approximation, $R_e \to \infty$, is made and we set $\phi_o = 0$, then $\chi = \phi$, $d\psi/d\hat{t} = (L/b)\hat{r}$, and $d\hat{y}/d\hat{t} = \beta + \psi$, where $\hat{y} = 2y_1/L$. Furthermore, the rest of the usual flat-Earth equations (Ref. 2 with $\gamma_e = 0$) follow from Eqs. (1).

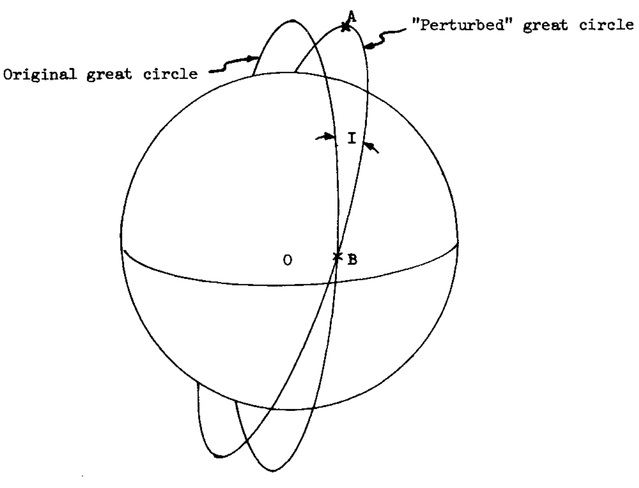

Fig. 2 Original great circle and that corresponding to the kinematical mode.

Conclusion

Linear perturbation equations for the lateral dynamics of flight on a great circle have been presented. The form of these equations is such that the vehicle dynamics are uncoupled from the flight path kinematics in a manner analogous to the corresponding flat-Earth equations. It has been shown that the new oscillatory mode cited in Ref. 1 corresponds to steady flight on a great circle which is, in general, not the reference flight path. In fact, for a dynamically stable vehicle, the steady-state motion following a perturbation will be motion in a "perturbed" great circle with the period of the new oscillatory mode.

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Finite-Difference Version of Quasi-Linearization Applied to Boundary-Layer Equations

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Introduction

The analysis of the boundary-layer equation is characterized by the split two-point boundary conditions. The feature presents an essential difficulty in numerical calculation of the boundary-layer problem. This difficulty appears both in similar flows, which are governed by one or more ordinary differential equations and in nonsimilar flows, which are governed by partial differential equations.

There have been a number of methods to overcome this difficulty. Among them, the quasi-linearization method developed by Radbill[1] seems to give a very powerful tool for treating the split boundary conditions. The method converts the nonlinear two-point boundary value problem into an iterative scheme of solution which involves the step-by-step integration of linear differential equations with two-point boundary conditions. The conditions at both boundaries are conserved and satisfied at every iteration.

In some circumstances, the finite-difference scheme is considered more desirable than the scheme of type of step-by-step integration. Lew presented an application of finite-difference scheme to boundary-layer equation.[2] His method consists of the accelerated replacement solution of simultaneous nonlinear algebraic equations. But the number of iteration cycles necessary for the prescribed criteria of the accuracy is rather large. Although using the acceleration parameter ω, in case of the two-dimensional stagnation point flow, for example, 47 iterations were necessary to obtain the solution within the given accuracy criterion $\varepsilon = 10^{-4}$. This seems the fatal point of the method.

In the present Note, we describe a finite-difference version of the quasilinearization method applied to boundary-layer equation. The method eliminates the estimation of the accelera-

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